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# SHORELINE PROFILE OF THE STOKES-MODE EDGE WAVES*

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Based on the assumptions of inviscid and incompressible fluid, irrotational flow, and infinitesimal wave amplitude, Stokes (1846) found a solution to the water-wave problem with a uniformly sloping impermeable boundary (beach). The solution, often termed the Stokes-mode edge wave, can be written in terms of velocity potential, $\phi$, as

$$\phi(x,y,z,t) = \frac{Ag}{\omega} \sin\beta\, e^{-ky\cos\beta + kz\sin\beta} \sin(kx - \omega t), \tag{1}$$

where A is the amplitude of wave runup distance along the beach surface, $\omega$ is the wave angular frequency, k is the wave number in the longshore direction, $\beta$ is the beach slope from the horizontal, and the coordinates (x, y, z) point to the alongshore, offshore, and vertically upward directions, respectively, as shown in Fig. 1. Equation (1) indicates that edge waves propagate parallel to the shoreline with their crests pointing offshore. The maximum wave amplitude occurs at the shoreline, $y = 0$, and decays exponentially offshore with an e-folding distance of $(k \cos\beta)^{-1}$. The corresponding dispersion relation is given by

$$\omega^2 = gk \sin\beta. \tag{2}$$

This dispersion relation indicates that edge waves are dispersive, i.e. the group velocity, $\partial\omega/\partial k$, is a function of k. Rotating the coordinates from the horizontal about the longshore axis by $\beta$, (1) and (2) can be expressed by

$$\phi(x,Y,t) = \frac{A\gamma}{\omega} e^{-kY} \sin(kx - \omega t), \tag{3}$$

and

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$$\omega^2 = \gamma k, \tag{4}$$

where Y is the coordinate pointing offshore along the beach surface as shown in Fig. 1, and $\gamma = g \sin \beta$ represents the down-slope component of gravity. With the rotation of coordinates, the Stokes-mode edge waves, (3), are no longer dependent on the coordinate Z perpendicular to the x-Y plane. Note that (3) and (4) have exactly the same forms as the linear solution of deep-water waves. It was shown that nonlinear behaviors of edge waves are also analogous to those of the two-dimensional deep-water waves (Yeh, 1987).

Just like the solution of deep-water waves, nonlinear correction in $\phi$ does not arise from the next higher-order solution of the Stokes-mode edge waves (Whitham, 1976). Nevertheless the nonlinear correction appears in the vertical surface displacement $\eta$. Substituting the linear solution (1) into the dynamic boundary condition along the free surface yields

$$\eta\,(x,y,t) = A \sin\beta\left[ e^{-ky\cos\beta} \cos(kx - \omega t) - \frac{1}{2} Ak\, e^{-2ky\cos\beta}\right] + O\left(A^3k^2; \beta^3\right), \tag{5}$$

where the beach slope $\beta$ is assumed to be small. The second term in parenthesis in (5) represents the set-down of mean water level of which effect decays exponentially offshore with an e-folding distance of $(2\,k \cos\beta)^{-1}$. The set-down effect of edge waves was verified in the laboratory experiments by Yeh (1986). While the vertical water surface displacement has the characteristics represented by (5), it is shown in this paper that the shoreline profile along the beach surface does not involve the set-down but instead the second-harmonic component appears at the same order of the solution.

Instead of the standard Eulerian formulation, edge-wave motions are first formulated in the Lagrangian coordinates with one of the axes pointing along the beach surface as shown in Fig. 1. Assuming an incompressible fluid, the equation of mass conservation can be written as

$$\frac{\partial(x, Y, Z)}{\partial(a, b, c)} = 1, \tag{6}$$

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where the operator $\frac{\partial(\cdot)}{\partial(\cdot)}$ is the Jacobian, and (a, b, c) are the Lagrangian coordinates to identify a fluid particle: the coordinates (a, b, c) are often taken to be the initial Cartesian coordinates of the fluid particle at t = 0, although such an assignment is not necessary. Assuming the inviscid fluid, the equations of motion in the (x, Y, Z) directions are, respectively,

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$$\frac{\partial^2 x}{\partial t^2} = -\frac{1}{\rho}\frac{\partial (p, Y, Z)}{\partial (a, b, c)},$$

$$\frac{\partial^2 Y}{\partial t^2} = -\frac{1}{\rho}\frac{\partial (x, p, Z)}{\partial (a, b, c)} + g \sin\beta, \tag{7}$$

$$\frac{\partial^2 Z}{\partial t^2} = -\frac{1}{\rho}\frac{\partial (x, Y, p)}{\partial (a, b, c)} - g \cos\beta,$$

with the boundary conditions,

$$Z = 0 \text{ at } c = 0,$$
$$p = 0 \text{ at } c = b \tan\beta, \tag{8}$$

where p is the pressure. An advantage of the Lagrangian formulations, (6) and (7), is that a moving runup waterline can be described with the independent variables (a, b, c), hence the location of the waterline is a known quantity *a priori*.

Using the straightforward (regular) perturbation method with the small parameter $\varepsilon = O(\tan^2\beta) = O(Ak)$, the second-order solution of the Stokes mode in the Lagrangian coordinate system is found to be

$$x = a - A e^{-kb} \sin(ka - \omega t) + O(\varepsilon^3),$$
$$Y = b - A e^{-kb} \cos(ka - \omega t) - \frac{1}{2} A^2 k e^{-2kb} + O(\varepsilon^3), \tag{9}$$
$$Z = c + O(\varepsilon^3),$$

where A is the wave runup amplitude along the beach surface. The shoreline profile is found by evaluating (9) at b = 0 and c = 0:

$$x = a - A \sin(ka - \omega t) + O(\varepsilon^3), \tag{10}$$

$$Y = -A \cos(ka - \omega t) - \frac{1}{2} A^2 k + O(\varepsilon^3). \tag{11}$$

The shoreline profile in the Eulerian-coordinate system can be retrieved by substituting (10) of the form,

$$a = x + A \sin(ka - \omega t) + O(\varepsilon^3), \tag{12}$$

into (11). Using some trigonometric identities and the Taylor series expansion, (11) becomes,

$$Y(b = 0; c = 0) = -A\left[\cos(kx - \omega t) + \frac{1}{2}Ak\cos(2kx - 2\omega t)\right] + O(A^3k^2), \qquad (13)$$

Compared with (5), the set-down effect in the water-surface profile, $\eta$, (the second term on the right-hand side of (5)) does not appear in the shoreline profile along the beach surface in (13); instead, the shoreline profile contains its second-harmonic component. Hence, the shoreline profile has features of the peaked runup and flattened rundown. Figures 2 a and b demonstrate the difference between the water-surface profile at the initial shoreline (5) and the shoreline profile along the beach surface (13).

Taking $\eta$ (at the shoreline) = - Y sin $\beta$ and solving (5) for Y also yield (13). Hence, (13) is consistent with (5). The second-harmonic component in the shoreline profile is due to the interference of the exponential offshore profile with the linearly sloping beach surface. This effect is demonstrated in Fig. 3. At Y = 0 (the shoreline location for the quiescent state), (5) predicts that the vertical water-surface elevation is sinusoidal in x and t. However, the value of $\eta$ cannot be negative at Y = 0, hence during the rundown phase, the shoreline is determined at the intersection of the exponential decay of the profile and the linear beach surface. During the runup phase, the shoreline is determined by the exponential extrapolation from Y = 0. The exponential extrapolation/interpolation is the one that causes the features of the second harmonic in the shoreline profile. The set-down effect in (5) occurs in order to satisfy the conservation of mass. This result may be important when field data of shoreline variations are analyzed for edge waves.

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Figure 2. -- Edge-wave profiles: a) water-surface elevation at y = 0 based on (5); b) shoreline profile based on (13). Ak = 0.5 for exaggeration of the nonlinear effects.

Figure 3. -- Maximum and minimum offshore profiles of the edge wave. Ak = 0.5 for exaggeration of the nonlinear effects.

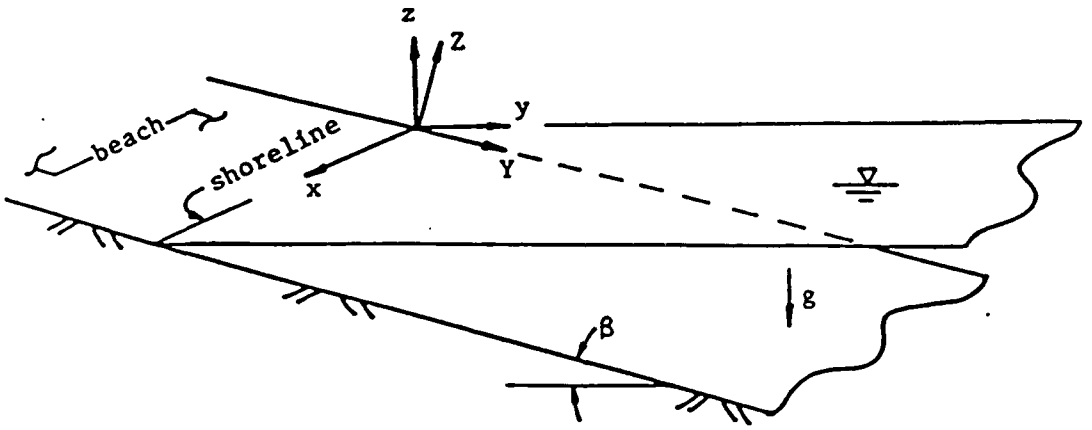


Fig. 1.

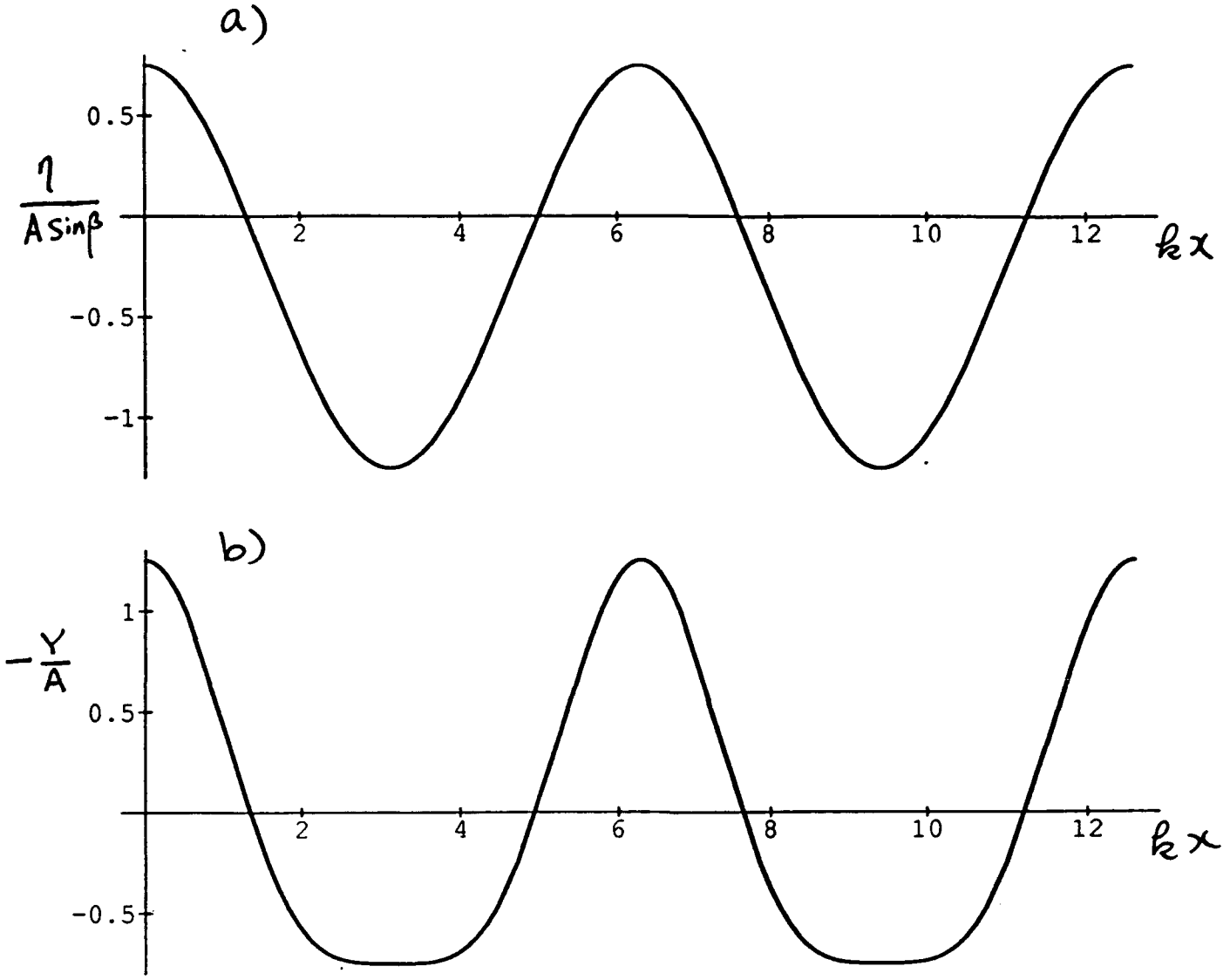
a)
0.5
$\frac{\eta}{A \sin\beta}$
2
4
6
8
10
12
kx
-0.5
-1
b)
1
$-\frac{Y}{A}$
0.5
2
4
6
8
10
12
kx
-0.5

Fig. 2

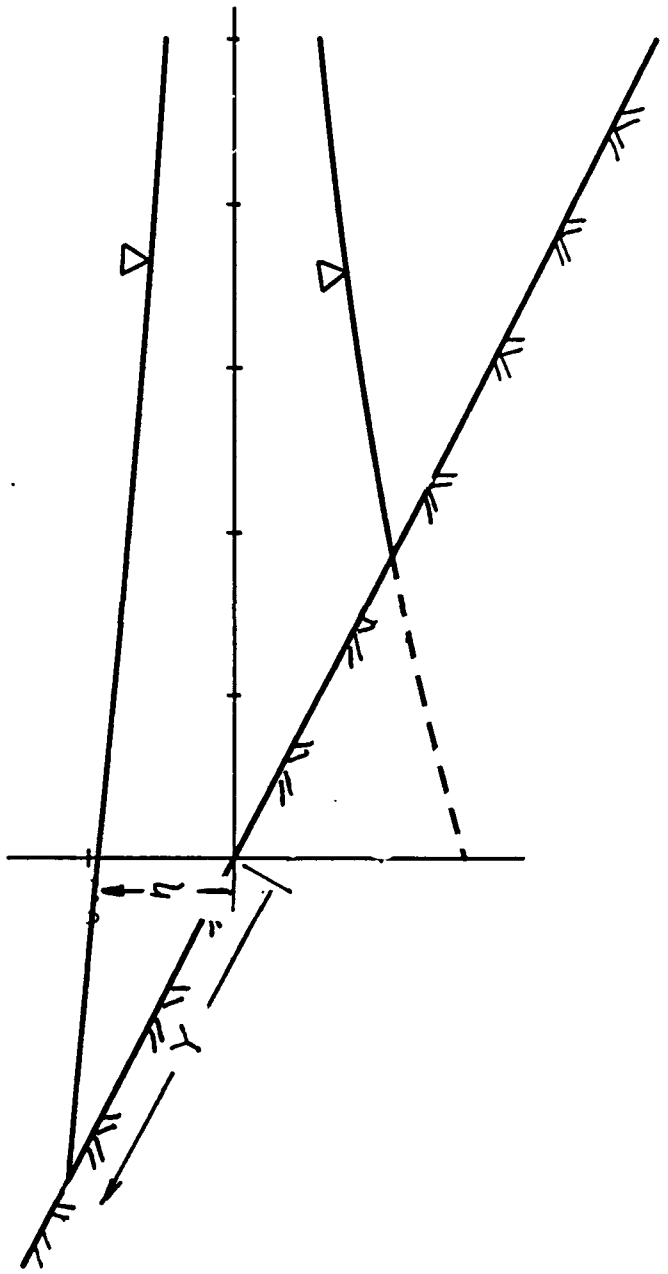

Fig. 3.